

Isospin violating decays of positive parity B_s mesons in $\text{HM}\chi\text{PT}$

Svjetlana Fajfer^{1,2,a}, Anita Prapotnik Brdnik^{2,3,b}

¹ Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia

² J. Stefan Institute, Jamova 39, P. O. Box 3000, 1001 Ljubljana, Slovenia

³ Faculty of Civil Engineering, Transportation Engineering and Architecture, University of Maribor, Smetanova ulica 17, 2000 Maribor, Slovenia

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Abstract Recent lattice QCD results suggest that the masses of the first two positive parity B_s mesons lie below the BK threshold, similar to the case of $D_{s0}^*(2317)^+$ and $D_{s1}(2460)^+$ mesons. The mass spectrum of B_s mesons seems to follow the pattern of a D_s mass spectrum. As in the case of charmed mesons, the structure of positive parity B_s mesons is very intriguing. To shed more light on this issue, we investigate the strong isospin violating decays $B_{s0}^{*0} \rightarrow B_s^0 \pi^0$, $B_{s1}^0 \rightarrow B_s^{*0} \pi^0$, and $B_{s1}^0 \rightarrow B_s^0 \pi \pi$ within heavy meson chiral perturbation theory. The two-body decay amplitude arises at tree level and we show that the loop corrections give significant contributions. On the other hand, in the case of three-body decay $B_{s1}^0 \rightarrow B_s^0 \pi \pi$ the amplitude occurs only at loop level. We find that the decay widths for these decays are $\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi \pi) \sim 10^{-3}$ keV, and $\Gamma(B_{s0}^{*0} \rightarrow B_s^0 \pi^0) \leq 55$ keV, $\Gamma(B_{s1}^0 \rightarrow B_s^{*0} \pi^0) \leq 50$ keV. More precise knowledge of the coupling constant describing the interaction of positive and negative parity heavy mesons with light pseudo-scalar mesons would help to increase the accuracy of our calculation.

1 Introduction

Two positive parity mesons B_s states: the $J^P = 1^+$ state $B_{s1}(5830)^0$ and the $J^P = 2^+$ state $B_{s2}^*(5840)^0$ were observed by the CDF and LHCb collaborations [1–4]. Recent lattice results [5,6], as well as other work [7–15], have indicated that the observed states are most likely members of the $(1^+, 2^+)$ doublet. However, the positive parity doublet of the B_s states $(0^+, 1^+)$ is still unobserved. The above-mentioned studies also suggest that the $(0^+, 1^+)$ doublet of B_s states might have masses below the BK and B^*K thresholds. However, some relativistic quark models analyses [16–18] suggested that masses of the $(0^+, 1^+)$ doublet B_s

states should be above the BK and B^*K thresholds. This reminds one strongly of the “history” of establishing the charm meson spectrum in which detection of positive parity states below DK threshold was not predicted by the quark models. After the observation of $D_{s0}^*(2317)$ and $D_{s1}(2460)$ charmed mesons, we faced a long-lasting dilemma on the structure of D_s $(0^+, 1^+)$. The issue is whether $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are $\bar{q}q$ states or more exotic compounds [14,15,19]. One of the first explanations of the low mass of $D_{s0}^*(2317)$ was offered by the authors of [13], based on the unitarized model of DK mesons scattering. Similarly, the work of [14] based on the scattering of Goldstone bosons off heavy-light pseudo-scalar and vector mesons with the use of the approximate crossing symmetry of the unitarized scattering amplitude, predicted the existence of positive parity B meson states. The masses of positive parity heavy mesons were studied within the heavy meson chiral Lagrangians approach by the authors of [15,20]. The dilemmas on the structure of positive parity charm meson states are nicely summarized in the work of [19].

It was already suggested by the authors of [19] that a study of the strong and radiative decay modes of positive parity D_s states might help in differentiating between these scenarios. Since both systems of positive parity states D_s and B_s are rather similar, the systematic analyses of strong and radiative decay dynamics of B_s mesons would help in clarifying their structure. In our study we rely on the results of lattice calculations presented in Ref. [5]. These authors determined the spectrum of B_s $1P$ states and they found that the masses of $B_{s1}(5830)^0$ and $B_{s2}^*(5840)^0$ agree very well with the experimental results [1–4]. They predicted also the existence of the spin zero positive parity state ($J^P = 0^+$) with the mass $m_{B_{s0}} = 5.711(13)(19)$ GeV and the state $J^P = 1^+$ with the mass $m_{B_{s0}} = 5.750(17)(19)$ GeV. Both states have masses below the BK and B^*K threshold. This immediately indicates that both states can decay strongly if isospin is violated. Motivated by the result of lattice calculation and

^a e-mail: svjetlana.fajfer@ijs.si

^b e-mail: anita.prapotnik@um.si

relying on our findings in the appropriate charm sector [21], we determine the partial decay widths of both meson states to the final state containing one or two pions: $B_{s0}^* \rightarrow B_s^0 \pi^0$, $B_{s1}^0 \rightarrow B_s^* \pi^0$, and $B_{s1}^0 \rightarrow B_s^0 \pi \pi$.

Studies of these decays were performed already by [10–12, 22–24]. The authors of [23] assumed that the positive parity 0^+ and 1^+ B_s states have a structure of BK molecules, accounting for the similarity with D_s , and they suggest that they are rather narrow states with partial decay widths about 50–60 keV. On the other hand, authors of [10–12, 22] used different approach based on the assumption that the decays proceed through the channels $B_{s0}^* \rightarrow B_s^0 \eta \rightarrow B_s^0 \pi^0$ and $B_{s1}^0 \rightarrow B_s^* \eta \rightarrow B_s^* \pi^0$ with the help of η – π mixing and predicted the partial decay widths in the range of 10–40 GeV. As already discussed in [20, 21, 25, 26] chiral loop corrections play an important role in strong decays of D_s positive parity states and their contribution to the strong decay modes can be as large as the effect of η – π mixing. Since π and $\pi\pi$ in the final state of these decays are having very small momenta, both decay modes are ideal to use heavy meson chiral perturbation theory (HM χ PT).

In this paper, we determine the isospin violating decay amplitudes of positive parity B_s mesons, members of the $(0^+, 1^+)$ doublet, using HM χ PT. For two-body decays, there is a tree-level contribution to decay amplitude arising from the η – π mixing and loop contribution which is the divergent. The divergent loop contribution requires the regularization by the counter-terms. On the other hand, in the isospin violating two-body decays of D_{s0}^* (2317) and D_{s1} (2460) mesons, chiral loops contribute significantly [21]. This was indicated already in Ref. [27] within a different framework in which only part of the loop contributions are included in the decay amplitudes of D_{s0}^* (2317) and D_{s1} (2460). As we pointed out in [21], the isospin violating three-body decay amplitude can arise at loop level only within HM χ PT. These loop contributions are then finite. In the case of charm decays, the ratio of the decay widths for $D_{s1}(2460)^+ \rightarrow D_s^{*+} \pi^0$ and $D_{s1}(2460)^+ \rightarrow D_s^+ \pi^+ \pi^-$ is known experimentally. From this ratio we were able to constrain the finite size of the counter-terms necessary to regularize the two-body decay amplitude $D_{s1}(2460)^+ \rightarrow D_s^{*+} \pi^0$. The heavy quark symmetry implies the same size of counter-term contributions for the B_s system as in the case of charm mesons. Therefore, by adopting the result of the lattice calculation showing that B_s mesons, part of the $(0^+, 1^+)$ doublet, have masses below BK and BK^* , we are able to predict their partial decay widths.

The basic HM χ PT formalism is introduced in Sect. 2. In Sect. Theampl we calculate the decay widths of the two-body strong decays of positive parity B_s doublet $(0^+, 1^+)$. In Sect. 4, the calculation of the three-body decay width $B_{s1}^0 \rightarrow B_s^0 \pi \pi$ decay mode will be presented, while a short conclusion will be given in Sect. 5.

2 Framework

In our analysis we rely on HM χ PT (see e.g. [28, 29]). This approach combines the heavy quark effective theory with the chiral perturbation theory and can be used to describe the decays of mesons that are composed of one light and one heavy quark. The chiral perturbation theory works very well in the case where the pseudo-scalar mesons have low momenta. In the heavy meson limit, heavy mesons, pseudo-scalar and vector, as well as scalar and axial, become degenerate. The negative parity states are described by the field H , while the positive parity states are entering in the field S :

$$H = \frac{1}{2}(1 + v \cdot \gamma)[P_\mu^* \gamma^\mu - P \gamma_5],$$

$$S = \frac{1}{2}(1 + v \cdot \gamma)[P_{1\mu}^* \gamma^\mu \gamma_5 - P_0], \quad (1)$$

where P_μ^* and P annihilate the vector and pseudo-scalar mesons, respectively, while $P_{1\mu}^*$ and P_0 annihilate the axial-vector and scalar mesons, respectively. Within chiral perturbation theory, the light pseudo-scalar mesons are accommodated into the octet $\Sigma = \xi^2 = e^{(2i\Pi/f)}$ with

$$\Pi = \begin{pmatrix} \pi^0/\sqrt{2} + \eta_8/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta_8/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta_8/\sqrt{6} \end{pmatrix} \quad (2)$$

and $f \sim 120$ MeV at one-loop level [30]. The leading order of the HM χ PT Lagrangian, which describes the interaction of heavy and light mesons, can be written as

$$\mathcal{L} = -Tr[\bar{H}_a(i v \cdot \mathcal{D}_{ab} - \delta_{ab} \Delta_H) H_b] + g Tr[\bar{H}_b H_a \gamma \cdot \mathcal{A}_{ab} \gamma_5] + Tr[\bar{S}_a(i v \cdot \mathcal{D}_{ab}) - \delta_{ab} \Delta_S] S_b + \tilde{g} Tr[\bar{S}_b S_a \gamma \cdot \mathcal{A}_{ab} \gamma_5] + h Tr[\bar{H}_b S_a \gamma \cdot \mathcal{A}_{ab} \gamma_5], \quad (3)$$

where $\mathcal{D}_{ab}^\mu = \delta_{ab} \partial^\mu - \mathcal{V}_{ab}^\mu$ is a heavy meson covariant derivative, $\mathcal{V}_\mu = 1/2(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$ is the light meson vector current and $\mathcal{A}_\mu = i/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ is the light meson axial current. A trace is taken over the spin matrices and repeated light quark flavor indices. All terms in (3) are of the order $\mathcal{O}(p)$ in the chiral power counting (see e.g. [26]). Following the notation of [5], $\Delta_{SH} = \Delta_S - \Delta_H = 375$ GeV, and in order to maintain a well-behaved chiral expansion, we consider that this difference is of the order of the pion momentum, $\Delta_{SH} \sim \mathcal{O}(p)$ as in [26].

Light mesons are described by the Lagrangian [28, 29], which is of the order $\mathcal{O}(p^2)$ in the chiral expansion,

$$\mathcal{L}_0 = \frac{f^2}{8} Tr[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + \frac{f^2 \lambda_0}{4} Tr[m_q \Sigma + \Sigma^\dagger m_q], \quad (4)$$

where $\lambda_0 = m_\pi^2/(m_u + m_d) = (m_{K^+}^2 - m_{K^0}^2)/(m_u + m_d) = (m_K^2 - m_\pi^2/2)/m_s$ and m_q is a diagonal quark matrix.

From the second term in (4), we can derive the η - π mixing Lagrangian [31,32]:

$$\mathcal{L}_{\eta-\pi_0} = \frac{m_\pi^2(m_u - m_d)}{\sqrt{3}(m_u + m_d)}\pi_0\eta. \quad (5)$$

The scalar (pseudo-scalar) and vector (axial-vector) heavy meson propagators can be written in the form:

$$\frac{i}{2(k \cdot v - \Delta_i)} \quad \text{and} \quad \frac{-i(g^{\mu\nu} - v^\mu v^\nu)}{2(k \cdot v - \Delta_i)}, \quad (6)$$

respectively, where Δ_i in the propagator represents the residual mass of the corresponding field. The residual masses are responsible for mass splitting of the heavy meson states. The difference Δ_{SH} splits the masses of positive and negative parity states. In addition, we also have a mass splitting between B_s and B states as well as a mass splitting between vector (axial-vector) and pseudo-scalar (scalar) fields. According to [33], the mass splitting between B_s and B states is 87 MeV, while the splitting between vector and pseudo-scalar states is 45 MeV. Since these splittings are much smaller than Δ_{SH} , they can be safely neglected.

The coupling constants g , h , and \tilde{g} were already discussed by several authors and determined by several methods [34–51]. We will use recent results of the lattice QCD: $g = 0.54(3)(^{+2}_{-4})$ [39], $\tilde{g} = -0.122(8)(6)$, and $h = 0.84(3)(2)$ [45]. The lattice study for h coupling is derived from the non-strange scalar/axial states and then SU(3) symmetry has been used to relate non-strange/strange systems. However, based on the experimental and phenomenological studies of the B meson positive parity states, it was found that these systems with and without strangeness are rather different. That indicates the possibility that the nature of the positive parity B_s system is not necessarily the same as in the case of positive parity B states, which can cause a difference in a value of the coupling constant h for these two systems. Furthermore, existing phenomenological determinations of the coupling constant h , based on light-cone sum rules [34,38,49] or HM χ PT approach [26], are also performed on the non-strange systems and predict smaller values of $h \in (0.5-0.6)$. Due to lack of lattice studies of the coupling constant h in the strange systems, we vary h in the range (0.5–0.95) in order to investigate its influence on decay widths.

Lattice results will also be used for the B_{s0}^* and B_{s1} masses, as well as Δ_{SH} [5]: $m_{B_{s0}} = 5,711(13)(19)$ GeV, $m_{B_{s1}} = 5.75(17)(19)$ GeV, and $\Delta_{SH} = 375(13)(19)$ MeV.

In order to absorb divergences coming from loop integrals, one needs to include counter-terms. Following [25,26] the counter-term Lagrangian can be written as

$$\mathcal{L}_{ct} = \lambda_1[\bar{H}_b\bar{H}_a(m_q^\xi)_{ba}] + \lambda'_1[\bar{H}_a\bar{H}_a(m_q^\xi)_{bb}] - \tilde{\lambda}_1[\bar{S}_b\bar{S}_a(m_q^\xi)_{ba}] - \tilde{\lambda}'_1[\bar{S}_a\bar{S}_a(m_q^\xi)_{bb}]$$

$$\begin{aligned} & + \frac{h\kappa'_1\lambda_0}{(4\pi f)^2} Tr[(\bar{H}S\gamma_\mu\mathcal{A}^\mu\gamma_5)_{ab}(m_q^\xi)_{ba}] \\ & + \frac{h\kappa'_3\lambda_0}{(4\pi f)^2} Tr[(\bar{H}S\gamma_\mu\mathcal{A}^\mu\gamma_5)_{aa}(m_q^\xi)_{bb}] \\ & + \frac{h\kappa'_5\lambda_0}{(4\pi f)^2} Tr[\bar{H}_a S_a \gamma_\mu \mathcal{A}_{bc}^\mu \gamma_5 (m_q^\xi)_{cb}] \\ & + \frac{h\kappa'_9\lambda_0}{(4\pi f)^2} Tr[\bar{H}_c S_a \gamma_\mu \mathcal{A}_{bc}^\mu \gamma_5 (m_q^\xi)_{ab}] \\ & + \frac{\delta'_2}{(4\pi f)^2} Tr[\bar{H}_a S_b i v \cdot \mathcal{D}_{bc} \gamma_\mu \mathcal{A}_{ca}^\mu \gamma_5] \\ & + \frac{\delta'_3}{(4\pi f)^2} Tr[\bar{H}_a S_b i \gamma_\mu \cdot \mathcal{D}_{bc}^\mu v \cdot \mathcal{A}_{ca}^\mu \gamma_5] + h.c. + \dots, \quad (7) \end{aligned}$$

where $m_q^\xi = (\xi m_q \xi - \xi^\dagger m_q \xi^\dagger)/2$ and $D_{bc}^\alpha A_{ca}^\beta = \partial^\alpha A_{ba}^\beta + [v^\alpha A^\beta]_{ba}$. At the given scale, the finite part of κ'_3 can be absorbed into the definition of h . Parameters λ'_1 and $\tilde{\lambda}'_1$ can be absorbed into the definition of heavy meson masses by a phase redefinition of H and S , while λ_1 and $\tilde{\lambda}_1$ split the masses of SU(3) flavor triplets of H_a and S_a [25,26]. Therefore, only contributions proportional to κ'_1 , κ'_9 , κ'_5 , δ'_2 , and δ'_3 will be explicitly included in the amplitudes.

3 The amplitudes and the decay widths of two-body decay modes

At the tree level, the $B_{s0}^{*0} \rightarrow B_s \pi^0$ and $B_{s1}^0 \rightarrow B_s^* \pi^0$ decays occur through η - π mixing as shown in Fig. 1. The decay widths can be written as

$$\Gamma = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi^2 \delta_{\text{mix}}^2, \quad (8)$$

where E_π and k_π are the energy and momenta of the outgoing pion and δ_{mix} is the η - π mixing angle [25],

$$\delta_{\text{mix}} = \frac{1}{2\sqrt{2}} \frac{m_u - m_d}{m_s - (m_u + m_d)/2} = \frac{-1}{87\sqrt{2}}. \quad (9)$$

This yields

$$\Gamma(B_{s1}^0 \rightarrow B_s^* \pi^0) = 16 \text{ keV}, \quad \Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0) = 18 \text{ keV}. \quad (10)$$

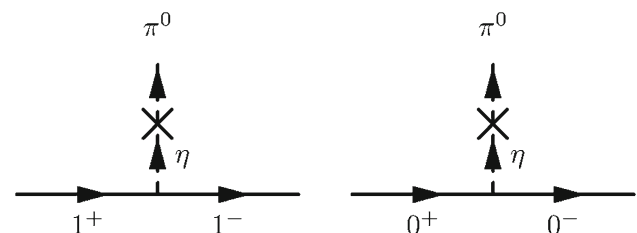
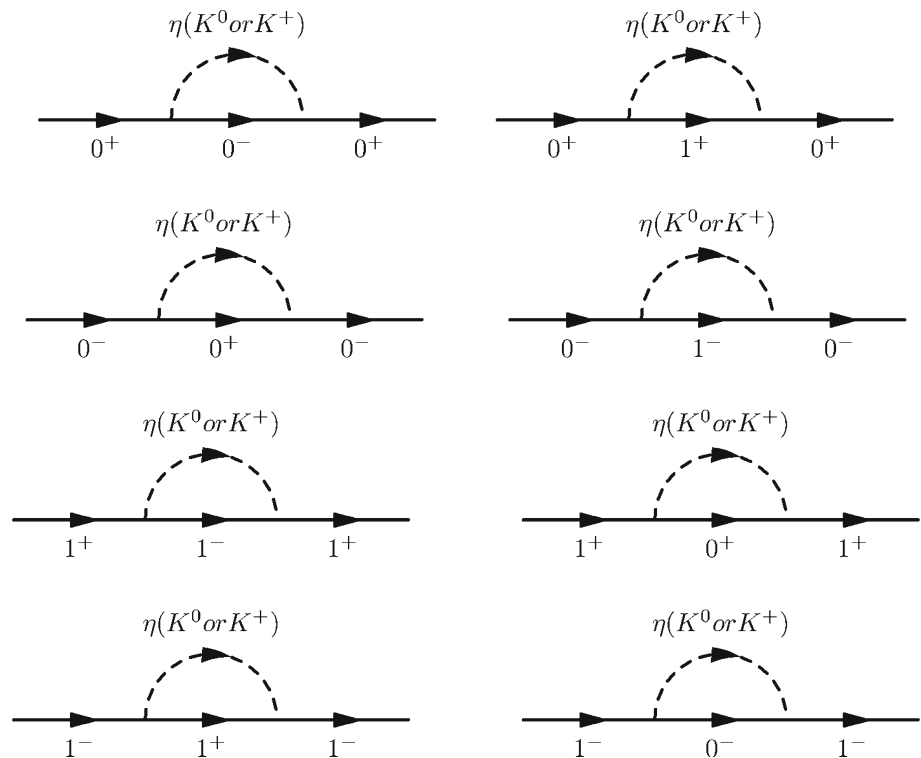


Fig. 1 Tree-level contribution to $B_{s0}^{*0} \rightarrow B_s \pi^0$ and $B_{s1}^0 \rightarrow B_s^* \pi^0$ decay modes

Fig. 2 Chiral corrections to the B mesons wave functions

By including chiral loop corrections, the decay width can be rewritten as

$$\Gamma = \frac{h^2}{2\pi f^2} |k_\pi| E_\pi^2 \delta_{\text{mix}}^2 \left| \frac{\sqrt{Z_{w,f} Z_{w,i}}}{Z_v} \right|^2, \quad (11)$$

where $Z_{w,f}$ and $Z_{w,i}$ denote the wave-function renormalization of the initial and final heavy meson states and Z_v represents the vertex corrections.

The wave-function renormalization factor is defined as

$$Z_{w,j} = 1 - \frac{1}{2} \frac{\partial \Pi_j(v \cdot p)}{\partial v \cdot p} \Big|_{\text{on mass shell}}, \quad (12)$$

where $\Pi_j(v \cdot p)$ is the meson self-energy calculated from the sunrise type diagrams in Fig. 2. For $Z_{w,j}$ we derive

$$Z_{w,j} = 1 - \mathcal{W}_j(m_{K^+}) - \mathcal{W}_j(m_{K^0}) - \frac{2}{3} \mathcal{W}_j(m_\eta), \quad (13)$$

with

$$\mathcal{W}_j(m_i) = \frac{1}{16\pi^2 f^2} \times (3\tilde{g}^2 \bar{B}'_{00}(0, m_i) - h^2 \bar{B}'_2(-\Delta_{SH}, -\Delta_{SH}, m_i)), \quad (14)$$

for the positive parity mesons and

$$\mathcal{W}_j(m_i) = \frac{1}{16\pi^2 f^2} \times (3g^2 \bar{B}'_{00}(0, m_i) - h^2 \bar{B}'_2(\Delta_{SH}, \Delta_{SH}, m_i)), \quad (15)$$

for the negative parity mesons. Here, \bar{B}'_{00} , \bar{B}'_2 are the Veltman–Pasarino loop integrals defined in Appendix A.

The vertex correction is defined as

$$Z_v = 1 - \frac{\hat{\Gamma}(v \cdot p_i, v \cdot p_f, k^2)}{\hat{\Gamma}_0(v \cdot p_i, v \cdot p_f, k^2)} \Big|_{\text{on mass shell}}. \quad (16)$$

Here $\hat{\Gamma}$ is the vertex amplitude calculated from the Feynman diagrams presented in Figs. 3 and 4, while $\hat{\Gamma}_0$ is the vertex amplitude resulting from the tree-level Feynman diagram (see Fig. 1):

$$Z_v = 1 - \left(\delta'_{\text{mix}} + \frac{2}{3} \mathcal{V}'(m_\eta) - \frac{1}{2} (\mathcal{V}(m_{K^+}) + \mathcal{V}(m_{K^0})) + \frac{1}{\sqrt{2} \delta_{\text{mix}}} (\mathcal{V}(m_{K^+}) - \mathcal{V}(m_{K^0})) + \mathcal{V}_{ct} \right), \quad (17)$$

where $\delta'_{\text{mix}} = 0.11$ includes corrections to the η – π mixing angle beyond tree level [25, 30], while \mathcal{V} and \mathcal{V}' are

$$\begin{aligned} \mathcal{V}(m_i) = & \frac{1}{16\pi^2 f^2} ((\bar{B}_{00}(-\Delta_{SH}, m_i) - \bar{B}_{00}(\Delta_{SH}, m_i) \\ & + \bar{B}_{11}(-\Delta_{SH}, m_i) - \bar{B}_{11}(\Delta_{SH}, m_i) \\ & - \Delta_{SH} \bar{B}_1(-\Delta_{SH}, m_i) - \Delta_{SH} \bar{B}_1(\Delta_{SH}, m_i))/2 \\ & - h^2 (\bar{B}'_{00}(-\Delta_{SH}, \Delta_{SH}, m_i) + \bar{B}'_{11}(-\Delta_{SH}, \Delta_{SH}, m_i)) \\ & + 3g\tilde{g} \bar{B}'_{00}(0, 0, m_i)), \end{aligned} \quad (18)$$

Fig. 3 Chiral corrections to the $B_{s1}^0 \rightarrow B_s^* \pi^0$ decay mode

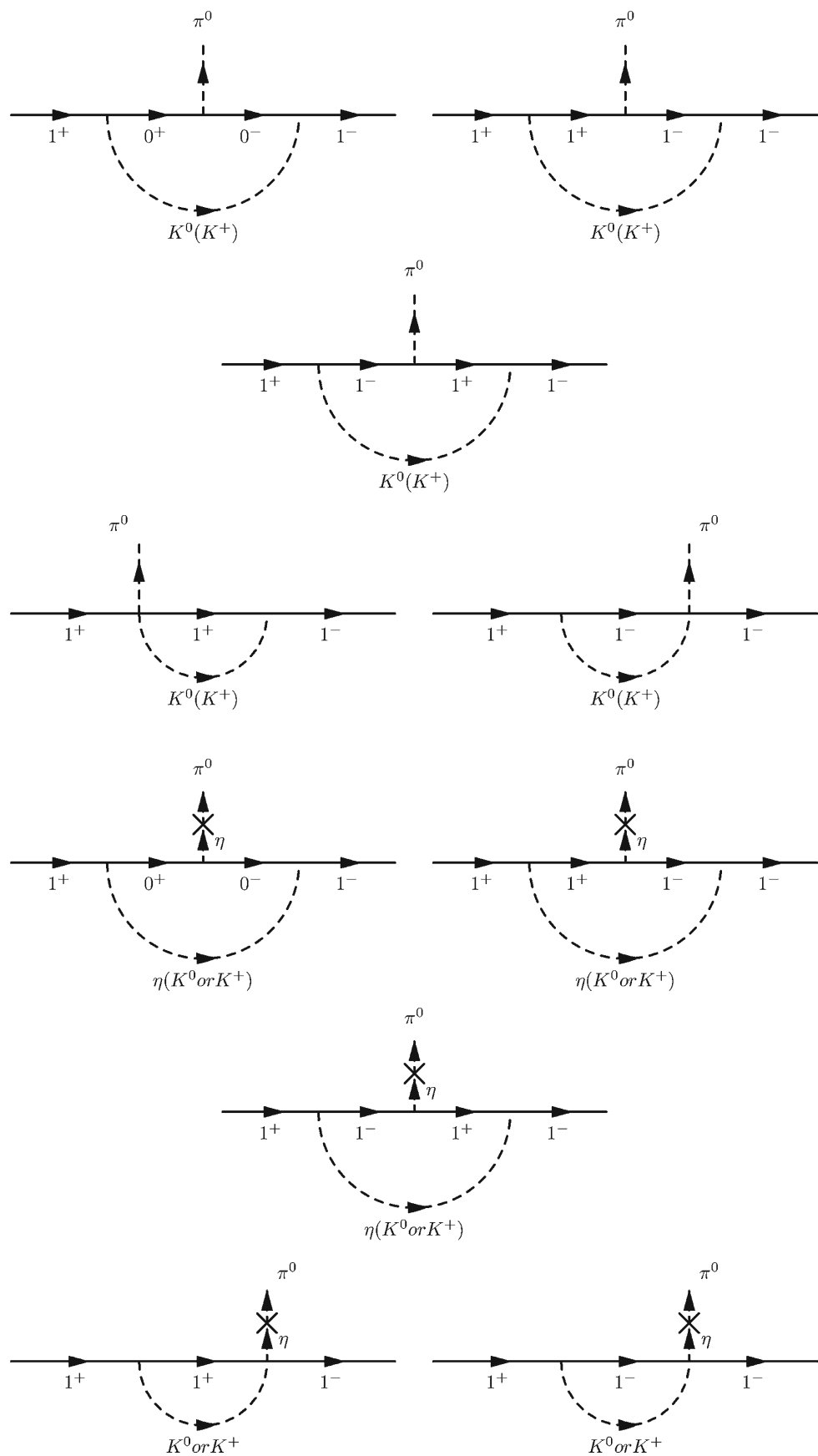
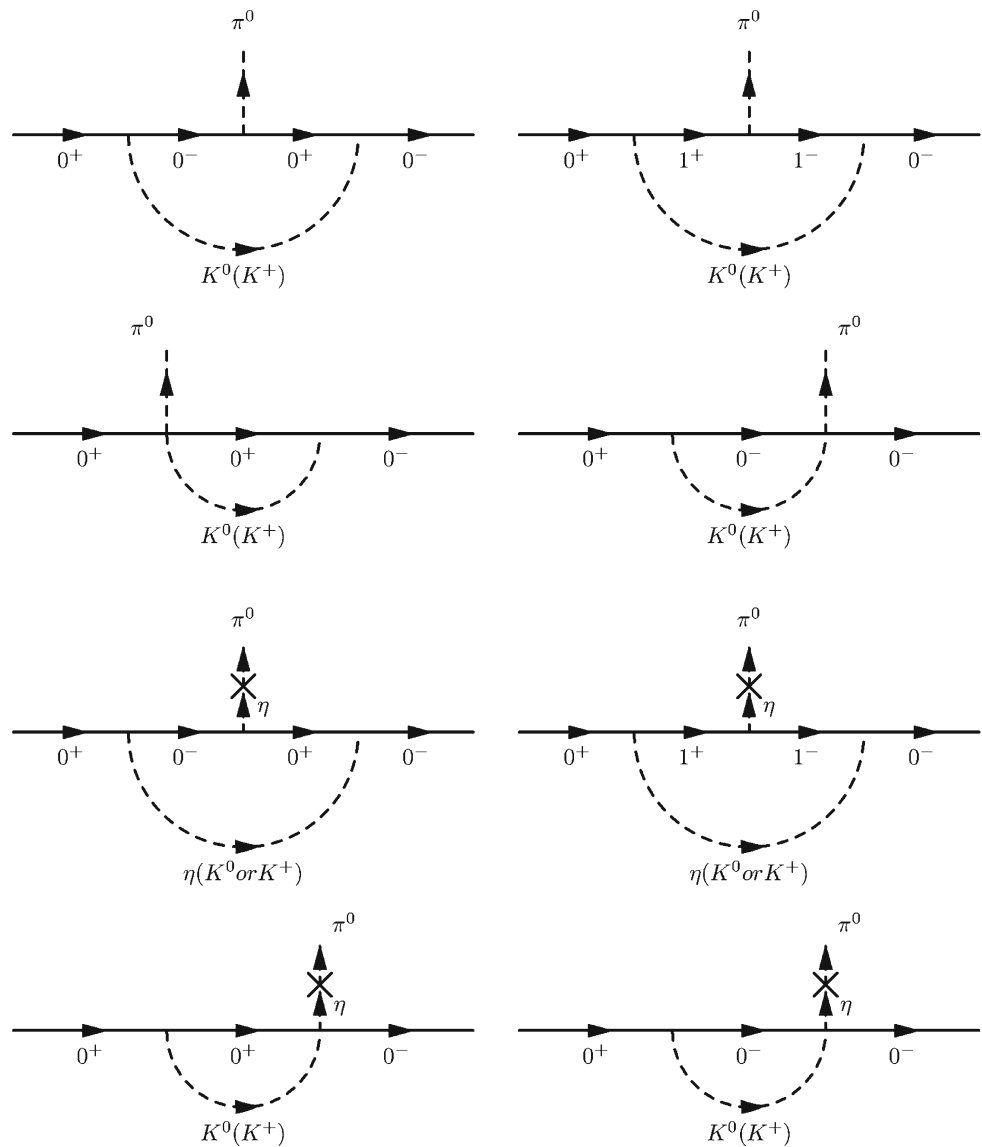


Fig. 4 Chiral corrections to the $B_{s0}^{*0} \rightarrow B_s \pi^0$ decay mode



$$\mathcal{V}'(m_i) = \frac{1}{16\pi^2 f^2} \left(-h^2 (\bar{B}'_{00}(-\Delta_{SH}, \Delta_{SH}, m_i) + \bar{B}'_{11}(-\Delta_{SH}, \Delta_{SH}, m_i)) + 3g\tilde{g}\bar{B}'_{00}(0, 0, m_i) \right). \quad (19)$$

Note that the isospin violating nature of both decay amplitudes manifests itself either by the proportionality of amplitude to the mixing parameter δ_{mix} , or by the mass difference $m_{K^0} - m_{K^+}$. Obviously in the isospin limit, the amplitudes vanish for $\delta_{\text{mix}} \rightarrow 0$ and $m_{K^0} = m_{K^+}$.

The finite parts of the counter-terms are collected in the term \mathcal{V}_{ct} :

$$\mathcal{V}_{ct} = \frac{1}{32\pi^2 f^2} \left(\left(m_K^2 - \frac{m_\pi^2}{2} \right) (\kappa'_1 + \kappa'_9) + \left(m_K^2 - m_\pi^2 + \frac{\sqrt{2}(m_{K^+}^2 - m_{K^0}^2)}{\delta_{\text{mix}}} \right) \kappa'_5 + \frac{E_\pi}{2\lambda_0} (\delta'_2 + \delta'_3) \right). \quad (20)$$

Neglecting the terms that are multiplied by m_π^2 and $\frac{E_\pi}{2\lambda_0}$ and by taking $m_{K^+}^2 = m_{K^0}^2$, all counter-terms can be replaced with the linear combination $\kappa' = \kappa'_1 + \kappa'_9 + \kappa'_5$, yielding

$$\mathcal{V}_{ct} = \frac{m_K^2}{32\pi^2 f^2} \kappa'. \quad (21)$$

Due to heavy meson symmetry, the same counter-term appears also in the case of D_s positive parity meson decays. In [21] we were able to constrain the size of this counter-term using the experimentally known ratio of the decay widths of the $D_{s1}(2460) \rightarrow D_s^* \pi$ and $D_{s1}(2460) \rightarrow D_s \pi \pi$ decay modes. The decay widths are also rather sensitive to the value of the coupling constant h as already noticed in [21], for the charm meson decays. The wave-function renormalization factor is responsible for this behavior.

The dependence of the decay widths on the coupling constant h in the range (0.5–0.95) is shown in Fig. 5. As seen

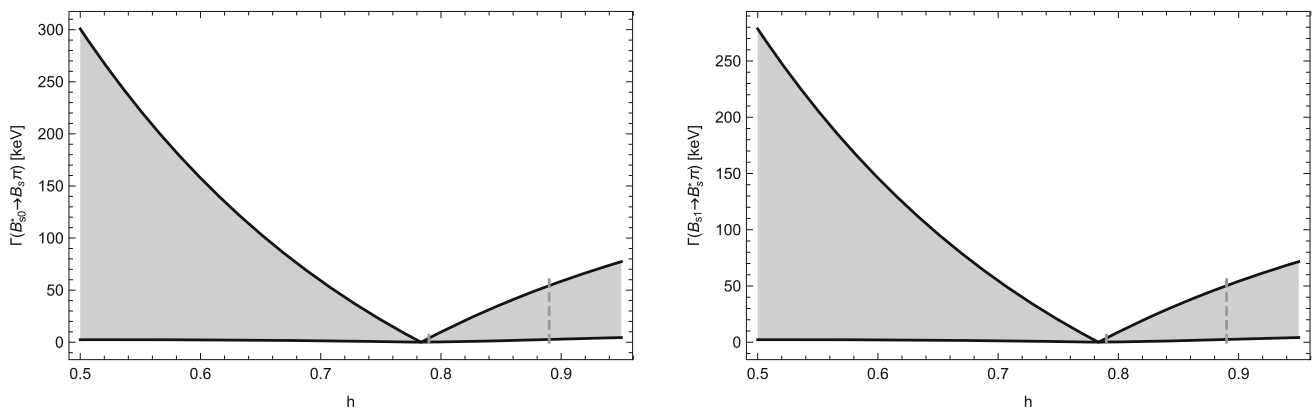


Fig. 5 Dependence of $\Gamma(B_{s1}^0 \rightarrow B_s^{*0} \pi^0)$ (right) and $\Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0)$ (left) on the coupling constant h

from Fig. 5, the decay widths are in the range of (0.1–55) keV for the range of coupling constant $h = 0.84(3)(2)$ as found by lattice calculation [45]. This value of h has been obtained in the non-strange system and one might expect that the value is not necessarily the same for positive parity mesons containing a strange quark. As can be seen from Fig. 5, if one would measure larger decay widths than the above stated ones, this would indicate lower values of the coupling constant h and possibly a more complicated structure of the positive parity B_s mesons. Nevertheless, due to the uncertainties in the values of the counter-terms, the allowed region for the decay widths grows rapidly if we lower the h . Note that we use the range of values for the counter-term (0.1–1.2) as found in [21]. Hopefully, measurements of the decay widths for positive parity D_s mesons, planned in the near future, would enable us to determine counter-terms more precisely and consequently to help in achieving a better precision of our predictions. For the central value $h = 0.84$, the range is $1 \text{ keV} \leq \Gamma(B_{s1}^0 \rightarrow B_s^{*0} \pi^0) \leq 30 \text{ keV}$. The decay rates for $B_{s1}^0 \rightarrow B_s^{*0} \pi^0$ and $B_{s0}^{*0} \rightarrow B_s \pi^0$ are almost equal, with the small difference due to the different masses of the final and initial B_s states.

Finally, we have checked the sensitivity of our results on the choice of the B_{s0}^{*0} and B_{s1} masses as well as Δ_{SH} . If we use the lattice results for the masses from Ref. [6] instead of [5], our results are modified by about 10 %.

4 The three-body decays: amplitudes and decay widths

In the case of B_{s1}^0 , a three-body decay $B_{s1}^0 \rightarrow B_s^0 \pi \pi$ is also possible. The $B_{s1}^0 \rightarrow B_s^0 \pi \pi$ decay width, averaged over the B_{s1}^0 polarizations, can be written as

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_i^3} |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2, \quad (22)$$

where M_i denotes the mass of B_{s1}^0 . If p_- and p_+ are the momenta of π^+ and π^- , respectively, and q is the momentum

of B_s^+ , then $dm_{12}^2 = (p_+ + p_-)^2$ and $dm_{23}^2 = (p_- + q)^2$. In the heavy quark limit $P^\mu = M_i v^\mu$, $q^\mu = M_f v^\mu$, and $\epsilon \cdot v = 0$, the amplitude is simplified to the following form:

$$\mathcal{M} = \mathcal{A} \epsilon \cdot (p_+ - p_-) = \mathcal{A} \epsilon \cdot \Delta p.$$

The non-vanishing Feynman diagrams that contribute to the amplitude \mathcal{A} are presented in Fig. 6. Note that all diagrams with η meson in the loop give a vanishing contribution, as already discussed in [21]. The amplitude \mathcal{A} can then be written as

$$\mathcal{A} = \frac{h\sqrt{M_i M_f}}{16\pi^2 f^4} (a_1 + a_2 + b_1 + b_2 + c_1 + c_2), \quad (23)$$

where parts of the amplitudes can be written as a linear combination of the Veltman–Passarino functions:

$$a_1 = \frac{g}{2} (\bar{B}_1(-\Delta_{SH}, m_{K^0}) - \bar{B}_1(-\Delta_{SH}, m_{K^+})), \quad (24)$$

$$a_2 = \frac{\tilde{g}}{2} (\bar{B}_1(\Delta_{SH}, m_{K^0}) - \bar{B}_1(\Delta_{SH}, m_{K^+})), \quad (25)$$

$$b_1 = 2g \left((\bar{B}_2'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^0}) - \Delta/2 \cdot \bar{B}_1'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^0})) - (\bar{B}_2'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^+}) - \Delta/2 \cdot \bar{B}_1'(-\Delta_{SH}, -\Delta_{SH}/2, m_{K^+})) \right), \quad (26)$$

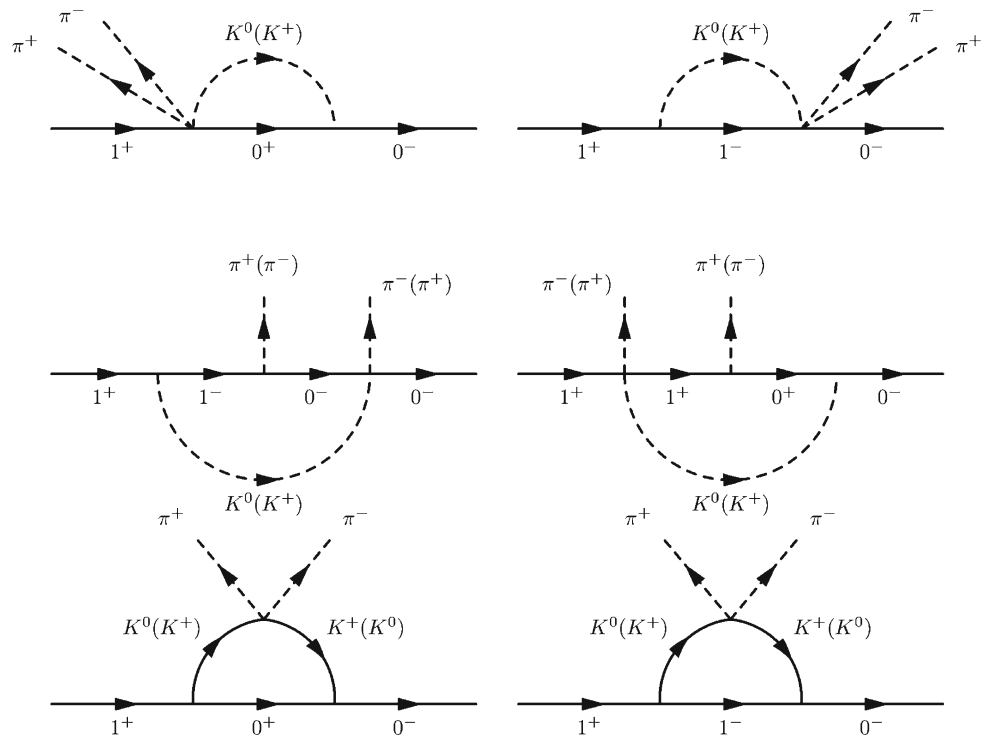
$$b_2 = 2\tilde{g} \left((\bar{B}_2'(\Delta_{SH}/2, \Delta_{SH}, m_{K^0}) + \Delta/2 \cdot \bar{B}_1'(\Delta_{SH}/2, \Delta_{SH}, m_{K^0})) - (\bar{B}_2'(\Delta_{SH}/2, \Delta_{SH}, m_{K^+}) + \Delta/2 \cdot \bar{B}_1'(\Delta_{SH}/2, \Delta_{SH}, m_{K^+})) \right), \quad (27)$$

$$c_1 = -2g \left((B_{00}(m_{K^0}) - \Delta_{SH} \bar{C}_{00}(-\Delta_{SH}, m_{K^0})) - (B_{00}(m_{K^+}) - \Delta_{SH} \bar{C}_{00}(-\Delta_{SH}, m_{K^+})) \right)$$

$$c_2 = -2\tilde{g} (B_{00}(m_{K^0}) - B_{00}(m_{K^+})). \quad (28)$$

Here, \bar{B}_1 , \bar{B}_2 , B_{00} , and \bar{C}_{00} are the Veltman–Passarino loop integrals defined in Appendix A. As the $B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-$ decay mode does not have any tree level contributions from the heavy meson Lagrangian, the amplitude is expected to be finite. Although some of the above integrals are divergent,

Fig. 6 Non-vanishing contributions to $B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-$ decay amplitude



these divergences cancel out as expected, when we take the sum of all contributions. We can also notice that the amplitude vanishes in the case of $m_{K^+} = m_{K^0}$, showing the nature of the isospin violating decay mode. The obtained decay widths are

$$\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi^+ \pi^-) = (1 \pm 0.3) \times 10^{-3} \text{ keV},$$

$$\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi^0 \pi^0) = (0.7 \pm 0.2) \times 10^{-3} \text{ keV}.$$

In the case of $B_{s1}^0 \rightarrow B_s^0 \pi^0 \pi^0$ a factor 1/2 was taken into account due to the two identical mesons in the final state.

5 Comments and discussion

Systematically using $\text{HM}\chi\text{PT}$, we determine the decay widths of the isospin violating decay modes of positive parity B_s mesons: $B_{s0}^{*0} \rightarrow B_s^0 \pi^0$, $B_{s1}^0 \rightarrow B_s^{*0} \pi^0$, and $B_{s1}^0 \rightarrow B_s^0 \pi \pi$. The masses of the decaying particles and the values of the coupling constants are taken from the lattice studies.

We find that the decay width $\Gamma(B_{s1}^0 \rightarrow B_s^0 \pi \pi) \sim 10^{-3} \text{ keV}$. This process occurs only at loop level and the decay amplitudes are proportional to the mass difference of K^+ and K^0 . The small available phase space additionally suppresses the decay width. This decay might also be approached by the exchange of the f_0 resonances $B_{s1}^0 \rightarrow B_s^0 f_0 \rightarrow B_s^0 \pi \pi$ [12]. However, in the $\text{HM}\chi\text{PT}$ this is a higher order contribution and therefore is not considered in our analysis. The approach of Ref. [12] (see Table 1) uses

the exchange of σ resonance in which there is a significant $\bar{s}s$ component. However, a recent lattice calculation of [52] disfavors such a content of σ .

The two-body decays of B_{s0}^{*0} and B_{s1}^0 occur at three level through η - π mixing. We find that the chiral loop corrections can significantly enhance or suppress the decay amplitudes being almost of the same order of magnitude as the tree-level contribution. We can only give a range of values for the decay widths. Namely, the decay widths are very sensitive to the value of the coupling constant h and change significantly if the coupling constant h is varied within the error bars determined by the lattice studies [45]. Also, the counter-terms are known only within a range of values in [21]. Let us note that the experimental measurement of the decay widths of the positive parity mesons $D_{s0}^{*0}(2317)^\pm$ and $D_{s1}(2460)^\pm$, planned in the near future by various experiments, can lead to a more precise determination of the counter-terms [21]. Together with a detection and a measurement of the decay widths and the branching ratios of the strong decays of the positive parity $(0^+, 1^+)$ B_s meson doublet, it will in principle allow one to extract the value of the coupling constant h exclusively from the D_s and B_s positive parity $(0^+, 1^+)$ doublet system. If this value would then deviate significantly from what is found by assuming the validity of $SU(3)$ symmetry or quark model considerations, it would point at an unusual nature of these states.

In Table 1, we give the results of other existing studies. The authors of [23] find higher values of the decay widths in the molecular picture of positive parity B_s states. In their

Table 1 Predictions of the $B_{s0}^{*0} \rightarrow B_s \pi^0$ and $B_{s1}^0 \rightarrow B_s^* \pi^0$ decay widths

Method	$\Gamma(B_{s0}^{*0} \rightarrow B_s \pi^0)$ (keV)	$\Gamma(B_{s1}^0 \rightarrow B_s^* \pi^0)$ (keV)	$\Gamma(B_{s1}^0 \rightarrow B_s \pi \pi)$ (keV)
Molecule picture [23]	46.7	50.1	
Molecule picture [27]	0.8 ± 0.8	1.8 ± 1.8	
Heavy quark and chiral symmetry [12]	21.5	21.5	≈ 0.05
Heavy chiral unitary approach [10, 11]	7.92	10.36	
3P_0 model [22]	35	38	
Light-cone sum rules [24]	7.92	10.36	
Chiral loop corrections (this work)			
$h = 0.79 - 0.89$	< 55	< 50	≈ 0.001

approach, however, wave-function renormalization, which in our case tends to lower the decay widths significantly, is not taken into consideration. Note also that the contributions of K^* loops, present in [23], are a higher order correction in the $\text{HM}\chi\text{PT}$ approach and therefore are not included in our analysis.

It will be interesting if current experimental searches at LHCb and planned studies at Belle II would lead to the discovery of both states B_{s0}^{*0} and B_{s1}^0 . We hope that our study might shed more light on this issue.

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Appendix A: Loop integrals

By employing dimensional regularization, in the renormalization scheme with $\delta = \frac{2}{4-D} - \gamma_E + \ln 4\pi + 1 = 0$, we have

$$\begin{aligned}
 A_0(m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)} = m^2 \left(\delta - \ln \frac{m^2}{\mu^2} \right) + \mathcal{O}(D-4), \\
 B_0(p, m, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)((k+p)^2 - m^2 + i\epsilon)} \\
 &= \delta - \int_0^1 \ln \frac{x^2 p^2 - x p^2 + m^2}{\mu^2} dx + \mathcal{O}(D-4), \\
 B_{00}(p, m, m) &= \frac{1}{2(D-1)} [A_0(m) + (2m^2 - p^2/2)B_0(p, m, m)],
 \end{aligned}$$

which in the $D \rightarrow 4$ limit gives

$$\begin{aligned}
 B_{00}(p, m, m) &= \frac{1}{6} [A_0(m) + (2m^2 - p^2/2)B_0(p, m, m) \\
 &\quad + 2m^2 - p^2/3],
 \end{aligned}$$

$$B_{00}(m) = B_{00}(\Delta_M v, m, m).$$

The loop integrals with one heavy meson propagator are

$$\begin{aligned}
 \bar{B}_0(\Delta, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times \int \frac{d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\
 &= -2\Delta \left[\delta - \ln \frac{m^2}{\mu^2} - 2F\left(\frac{m}{\Delta}\right) + 1 \right] + \mathcal{O}(D-4),
 \end{aligned}$$

with

$$F(1/x) = \begin{cases} \frac{1}{x} \sqrt{x^2 - 1} \ln(x + \sqrt{x^2 - 1} + i\epsilon); & |x| > 1, \\ \frac{-1}{x} \sqrt{1 - x^2} \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{x}{\sqrt{1 - x^2}} \right) \right); & |x| \leq 1, \end{cases}$$

$$\begin{aligned}
 \bar{B}^\mu(\Delta, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times \int \frac{k^\mu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\
 &= \bar{B}_1(\Delta, m) v^\mu, \\
 \bar{B}_1(\Delta, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times \int \frac{k \cdot v d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\
 &= A_0(m) + \Delta \bar{B}_0(\Delta, m), \\
 \bar{B}^{\mu\nu}(\Delta, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\
 &\times v \int \frac{k^\mu k^\nu d^D k}{(k^2 - m^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\
 &= \bar{B}_{00}(\Delta, m) g^{\mu\nu} + \bar{B}_{11}(\Delta, m) v^\mu v^\nu, \\
 \bar{B}_{00}(\Delta, m) &= \frac{1}{D-1} [(m^2 - \Delta^2) \bar{B}_0(\Delta, m) - \Delta A_0(m)],
 \end{aligned}$$

which in $D \rightarrow 4$ gives

$$\bar{B}_{00}(\Delta, m) = \frac{1}{3}[(m^2 - \Delta^2)\bar{B}_0(\Delta, m) - \Delta A_0(m) + 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_{11}(\Delta, m) = \frac{1}{D-1}[(D\Delta^2 - m^2)\bar{B}_0(\Delta, m) + D\Delta A_0(m)],$$

which in $D \rightarrow 4$ gives

$$\bar{B}_{11}(\Delta, m) = \frac{1}{3}[(4\Delta^2 - m^2)\bar{B}_0(\Delta, m) + 4\Delta A_0(m) - 2\Delta/3(3m^2 - 2\Delta^2)],$$

$$\bar{B}_2(\Delta, m) = \bar{B}_{00}(\Delta, m) + \bar{B}_{11}(\Delta, m),$$

$$\begin{aligned}\bar{B}'_0(\Delta_1, \Delta_2, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} \\ &= \frac{1}{\Delta_1 - \Delta_2}[\bar{B}_0(\Delta_1, m) - \bar{B}_0(\Delta_2, m)],\end{aligned}$$

$$\begin{aligned}\bar{B}^{\mu\nu}(\Delta_1, \Delta_2, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k^\mu d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} \\ &= \bar{B}'_1(\Delta_1, \Delta_2, m)v^\mu,\end{aligned}$$

$$\begin{aligned}\bar{B}'_1(\Delta_1, \Delta_2, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k \cdot v d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} \\ &= \bar{B}_0(\Delta_2, m) + \Delta_1 \bar{B}'_0(\Delta_1, \Delta_2, m),\end{aligned}$$

$$\begin{aligned}\bar{B}'_2(\Delta_1, \Delta_2, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{(k \cdot v)^2 d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} \\ &= A_0(m) + (\Delta_1 + \Delta_2)\bar{B}_0(\Delta_2, m) + \Delta_1^2 \bar{B}'_0(\Delta_1, \Delta_2, m),\end{aligned}$$

$$\begin{aligned}\bar{B}^{\mu\nu'}(\Delta_1, \Delta_2, m) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k^\mu k^\nu d^D k}{(k^2 - m^2)(v \cdot k - \Delta_1)(v \cdot k - \Delta_2)} \\ &= \bar{B}'_{00}(\Delta_1, \Delta_2, m)g^{\mu\nu} + \bar{B}'_{11}(\Delta_1, \Delta_2, m)v^\mu v^\nu,\end{aligned}$$

$$\begin{aligned}\bar{B}'_{00}(\Delta_1, \Delta_2, m) &= \frac{1}{D-1}[m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) \\ &- \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) - \bar{B}_1(\Delta_2, m)],\end{aligned}$$

which in $D \rightarrow 4$ gives

$$\begin{aligned}\frac{1}{3}[m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) - \Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) \\ - \bar{B}_1(\Delta_2, m) + 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1 \Delta_2))],\end{aligned}$$

$$\begin{aligned}\bar{B}'_{11}(\Delta_1, \Delta_2, m) &= \frac{1}{D-1}[-m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) \\ &+ D\Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) + D\bar{B}_1(\Delta_2, m)],\end{aligned}$$

which in $D \rightarrow 4$ gives

$$\begin{aligned}\frac{1}{3}[-m^2 \bar{B}'_0(\Delta_1, \Delta_2, m) + 4\Delta_1 \bar{B}'_1(\Delta_1, \Delta_2, m) \\ + 4\bar{B}_1(\Delta_2, m) - 2/3(3m^2 - 2(\Delta_1^2 + \Delta_2^2 + \Delta_1 \Delta_2))]\end{aligned}$$

The loop integrals with two heavy meson propagator are

$$\begin{aligned}\bar{C}^\mu(p, \Delta, m_1, m_2) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k^\mu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\ &= \bar{C}_1(p, \Delta, m_1, m_2)v^\mu,\end{aligned}$$

$$\begin{aligned}\bar{C}_1(p, \Delta, m_1, m_2) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k \cdot v d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\ &= B_0(p, m_1, m_2) + \Delta \bar{C}_0(p, \Delta, m_1, m_2),\end{aligned}$$

$$\begin{aligned}\bar{C}^{\mu\nu}(p, \Delta, m_1, m_2) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{k^\mu k^\nu d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)} \\ &= \bar{C}_{00}(p, \Delta, m_1, m_2)g^{\mu\nu} + \bar{C}_{11}(p, \Delta, m_1, m_2)v^\mu v^\nu,\end{aligned}$$

$$\begin{aligned}\bar{C}_{00}(\Delta, m) &= \bar{C}_{00}(-\Delta_M v, \Delta, m, m) \\ &= \frac{1}{D-1}[\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta)B_0(\Delta_M v, m, m) \\ &\quad + (m^2 - \Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m)],\end{aligned}$$

which in $D \rightarrow 4$ gives

$$\begin{aligned}\bar{C}_{00}(\Delta, m) &= \bar{C}_{00}(-\Delta_M v, \Delta, m, m) \\ &= \frac{1}{3}[\bar{B}_0(-\Delta_M + \Delta, m) - (\Delta_M/2 + \Delta)B_0(\Delta_M v, m, m) \\ &\quad + (m^2 - \Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m) - 2/3(3/2\Delta_M - \Delta)], \\ \bar{C}_{11}(\Delta, m) &= \bar{C}_{11}(-\Delta_M v, \Delta, m, m) \\ &= \frac{1}{D-1}[-\bar{B}_0(-\Delta_M + \Delta, m) + D(\Delta_M/2 + \Delta)B_0(\Delta_M v, m, m) \\ &\quad - (m^2 - D\Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m)],\end{aligned}$$

which in $D \rightarrow 4$ gives

$$\begin{aligned}\bar{C}_{11}(\Delta, m) &= \bar{C}_{11}(-\Delta_M v, \Delta, m, m) \\ &= \frac{1}{3}[-\bar{B}_0(-\Delta_M + \Delta, m) + 4(\Delta_M/2 + \Delta)B_0(\Delta_M v, m, m) \\ &\quad - (m^2 - 4\Delta^2)\bar{C}_0(\Delta_M v, \Delta, m, m) + 2/3(3/2\Delta_M - \Delta)].\end{aligned}$$

The calculation of the integral leaves us with

$$\begin{aligned}\bar{C}_0(p, \Delta, m_1, m_2) &= \frac{(2\pi\mu)^{4-D}}{i\pi^2} \\ &\times \int \frac{d^D k}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)(v \cdot k - \Delta + i\epsilon)};\end{aligned}$$

it is done in [53]. For some calculations, we used the program FeynCalc [54].

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